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The definability of physical concepts

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ABSTRACT: Our main purpose here is to make some specific considerations about the definability of physical concepts like mass, force, time, space, spacetime, and closed systems in the context of physical theories. Our starting motivation is a simple example of a collection of definitions of closed system in the literature of physics and philosophy of physics. Next we discuss the problem of definitions in theoretical physics from the point of view of modern theories of definition.

Keywords: definitions, foundations of physics, axiomatic method.

Contents

1. Introduction

This paper is some sort of continuation of a previous work published in this journal [25]. Some contents of both papers (mainly those concerning the classification of definitions in mathematics and physics) can be found in [26].

In a recent paper Sungho Choi [4] discussed the theory of conserved quantities in physics. He criticized Christopher Hitchcock's [14] definition of closed system, which is a strategic concept in order to cope with questions regarding conserved quantities. According to Hitchcock, a closed system is a system that does not engage in any *causal* interaction. In other words, a closed system is a system that does not interact with anything outside itself. In Choi's opinion, this kind of point is advocated by other authors of standard textbooks on physics like, e.g., [18]. But Choi thinks that this definition, which appeals to the notion of causality, entails circularity, since causal processes and interactions are analyzed in terms of conserved quantities that are in turn defined as physical quantities governed by conservation laws. So, conservation laws cannot invoke causation.

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Hence, he introduced a new definition for closed systems which is supposed to avoid circularity. Other advantages of this new definition are presented in his paper.

Choi's paper is obviously interesting and provocative. But we would like to make some remarks and criticisms and discuss some alternative ideas to his approach as a starting motivation for the present paper. First, the author raises very important questions about the foundations of physics, but a formal approach, by means of the axiomatic method, would be more appropriate, since the formalism in the axiomatic method allows us a more objective discussion about definitions and their real role in mathematics and in theoretical physics. We do advocate the use of formal systems if we want to talk about the notion of definability in theoretical physics. The presence of mathematics in physics is not a mere coincidence; it is a necessity. We believe that a formal approach to the problem of definability in theoretical physics can give another taste to this subject, since definitions play an important role in formal systems, at least from the logical point of view. If philosophy of science is supposed to deal with questions regarding the foundations of physics (among other subjects), then we should think about the possibility of using mathematics as one (although not the only one) common language among scientific philosophers. In this sense we completely agree with Patrick Suppes [32] about the role of the scientific philosopher nowadays; according to him:

> We are no longer Sunday's preachers for Monday's scientific workers, but we can participate in the scientific enterprise in a variety of constructive ways. Certain foundational problems will be solved better by philosophers than by anyone else. Other problems of great conceptual interest will really depend for their solution upon scientists deeply immersed in the discipline itself, but illumination of the conceptual significance of the solutions can be a proper philosophical role.

In order to make our first point clearer, we recall a similar question regarding the definability of concepts like mass and force in classical mechanics. As it is well known, some late XIX century European physicists like Heinrich Hertz and Gustav Kirchhoff were deeply concerned with the definability of the concept of force in mechanics [15,24]. On the other hand Ernst Mach tried to give the guidelines for a notion of inertial mass [27]. According to Max Jammer [15],

> [a]ny proof of the undefinability of mass in terms of other primitive notions can, of course, be given *only* within the framework of an axiomatization of mechanics.

The emphasis in the quote is ours. We would like to rephrase Jammer's idea as another more generalized statement. We consider that *any proof of either the formal undefinability or the formal definability of physical concepts like mass, force, time, space, closed systems, subsystems, etc., can be given* only *in the framework of an axiomatic system*. Our justification for this is as follows: the axiomatic framework is more objective than an intuitive approach. For example, if we define a closed system as a system that does not engage in any causal interactions, like it was proposed by Hitchcock [14], then a lot of questions should remain, mainly with respect to the meaning of each terminology used. For example, what is supposed to mean a system that does not engage in any causal interactions? Does that mean that the resultant external force over the system is null? If that is the case, then *there is* a causal interaction, although this causal interaction is null. Does that mean that there is no external force? If that is the case, how can we identify this system as a physical theory? In other words, we insist on the point of clarifying ideas by means of formal languages. What is a causal interaction? What is "to engage in a causal interaction"? What is a physical system?

We recognize that axiomatic systems represent a great deal of restriction on intuitive ideas, causing a loss of generalization in a sense. We do not think, e.g., that any physicist would recognize any axiomatic system for classical particle mechanics as a faithful picture of all the intuitive ideas concerning Newtonian particle mechanics. Besides, it is usually unclear what does it mean the label "Newtonian mechanics". Is Newtonian mechanics the subject that we find in some textbooks like [9]? Is Newtonian mechanics the set of ideas once stated by Isaac Newton in his famous *Principia*? That is one of the reasons why there is so many different axiomatic systems for physical theories. A physical theory is more than a mathematical structure or a bunch of axioms. A physical theory is always committed to experimental data and epistemological issues. Sometimes, physical theories are committed to ontological issues as well. Nevertheless, if someone intends to talk about the definability of physical concepts like, e.g., closed system, then a very careful description should be given to the notion of *definition*. There are many kinds of definition in the literature. But formal definitions refer only to formal languages, which are closely related to the axiomatic method. If a researcher intends to work with informal definitions, then this researcher should be very careful to answer the following question: what theory of definition is he/she using? What is a definition at all?

Our second point is that authors like Choi should discuss in their papers why circularity is a problem in a definition, since some authors defend the idea that circularity may be legitimate in some kinds of definition [11]. It is important to recall that circularity, from the point of view of some formal theories of definition, entails the non-eliminability of the *definiendum*. In other words, what kind of definitions do we intend to talk about? If circularity is an issue, what are other important questions regarding definitions? Some authors consider that all definitions should be noncreative in the sense that a definition should not allow a formula F as a theorem if this formula is not a theorem before the definition was stated. In what sense that a given definition is more appropriate than another one?

Finally, another issue seems to be somehow more difficult to deal with, in the case of Choi's paper. His definition of closed system makes an explicit use of the the notion of time. According to him, a system is closed with respect to a physical quantity Q at a time t if and only if either

$$\frac{dQ_{in}}{dt} = \frac{dQ_{out}}{dt} = 0$$

at t or,

$$\frac{dQ_{in}}{dt} \neq \frac{-dQ_{out}}{dt}$$

at t, where Q_{in} is the amount of Q inside the system and Q_{out} is the amount of Q outside the system. In the case of physical quantities represented as vectors or tensors, all you have to do is to consider that a system is closed if all the components of the vector or of the tensor satisfy one of the two conditions given above.

But what if there is any kind of fundamental relationship between time and causality? In other words, how to prove that Choi's definition of closed system is not circular at all? If terminology is not settled in a very formal way, any discussion like this seems to be vague, although it may excite some people in an intuitive manner. All strategic terms need to be formally settled.

Following this introduction, the next section presents a very brief review of the problem of definitions in some fields of scientific knowledge. In the third section Padoa's method is discussed. Padoa's method is a logical technique that allows us to decide if some kinds of concepts are definable. In the next section we use a well known axiomatization of classical particle mechanics in order to illustrate our ideas. Further discussions are made in the last sections.

2. Definitions in Science

Definitions in science have one major role, according to a widespread point of view; they allow us to introduce new terminologies in formal and natural languages. Such new terminologies are, in a precise sense, eliminable, dispensable, superfluous. For example, if we define the notion of human being as a political animal, then the statement "The human being is weird" may be rephrased as "The political animal is weird". In other words, the *definiendum* "human being" may always be replaced by the *definiens* "political animal". This illustration may cause false impressions, since some definitions are contextual and not explicit. In contextual definitions, the replacement of the *definiendum* by the *definiens* is not so straightforward, since it depends on the context of the statement where the replacement takes place.

Some authors have tried to present a general classification of definitions [10,22, 23]. Here we introduce a very brief classification at our own risk. We consider that every definition introduces new terminologies to a language and that the defined terminologies are always dispensable. If a new defined terminology is somehow related (in a non-trivial manner) to a formal language, then the definition is formal. Otherwise, we say that the definition is informal.

Informal definitions are very common in natural languages like English, for example. But we are mainly concerned here with formal definitions.

There are at least three kinds of formal definitions: ampliative, abbreviative, and Tarskian. An ampliative definition is that one that *adds* new symbols to a formal language. A well known example of ampliative theory of definitions is Leśniewski's [31]. According to Stanislaw Leśniewski, every definition is supposed to be eliminable and non-creative. The criterion of eliminability says that the *definiendum* can always be replaced by the *definiens* in any formula of the formal language. There is a close relationship between eliminability and circularity. If a given formula is circular, then it is not eliminable; so, it is not a definition at all, at least from a classical point of view. For details see [31]. The criterion of non-creativity says that a definition cannot allow the demonstration of new theorems that were impossible to prove before the definition was stated. This is a very difficult criterion, since it seems to be some kind of utopia. One example of a new result in a formal consistent framework would be a contradiction. But since there is no general method to verify if a theory is consistent, then the verification of the criterion of non-creativity seems to represent a task that in many cases is impossible to perform. So, it is not an easy task to verify if a formula is really a definition.

An abbreviative definition is that one that uses a metalinguistic symbol in order to *abbreviate* a sequence of symbols from a formal language [23]. For example, if we want to define the existential quantifier in first order languages by means of the universal quantifier, we do not need to add the symbol "$\exists$" to the language. We can consider the statement "$\exists x(F)$" as a metalinguistic abbreviation of the well-formed formula "$\neg\forall x(\neg F)$". We are obviously using standard notation for first-order languages, where the symbol $\neg$ corresponds to the logical connective of negation.

Finally, a Tarskian definition allows us to define sets in set-theoretical structures that are used as interpretations of formal languages [16,34]. If, e.g., Λ denotes a first order language and M is a set-theoretical interpretation of Λ, then a set X of M is definable if and only if there exists a well-formed formula $\varphi(y)$ in Λ with a single free occurrence of a variable y such that $x \in X$ iff x satisfies the formula $\varphi(y)$. In this case, the definability of sets in a set-theoretical structure depends on the language Λ. For details see the references.

Among the ampliative definitions we can still find two other kinds of definitions, namely, semantic and synthatic. A semantic definition is that one that adds a new symbol to a formal language by means of a metalinguistic symbol like the usual "$=_{def}$". For example, we can define the biconditional $\Leftrightarrow$ in first-order languages as

$$(A \Leftrightarrow B) =_{def} (A \Rightarrow B) \wedge (B \Rightarrow A), \tag{1}$$

where standard notation is used for logical connectives $\wedge$ and $\Rightarrow$.

A synthatic definition is that one which plays the role of an eliminable and noncreative axiom in an axiomatic framework. In this very bizarre situation, all definitions are new axioms. So, any new definition corresponds actually to the "definition" of a new theory. Some examples of this are given in [31].

In the case of semantic synthatic definitions

Informal definitions may be classified in many different ways. We do not intend to elaborate on this difficult topic. We can just say that there are operational definitions [12], ostensive definitions [10], and many others. So, we have the next diagram to summarize our ideas for the sake of our purposes:

$$
\text{Definitions}\left\{\begin{array}{l}
\text{Formal}\left\{\begin{array}{l}
\text{Abbreviative}\\
\text{Ampliative}\left\{\begin{array}{l}\text{Semantic}\\ \text{Synthatic}\end{array}\right.\\
\text{Tarskian}
\end{array}\right.\\
\text{Informal}\left\{\begin{array}{l}
\text{Operational}\\
\text{Ostensive}\\
\text{Explicit}\\
\text{Contextual}\\
\text{Other kinds}
\end{array}\right.
\end{array}\right.
$$

3. Padoa's method

Summarizing the previous discussion, in an axiomatic system S a primitive term or concept c is definable by means of the remaining primitive concepts if and only if there is an appropriate formula, provable in the system, that fixes the meaning of c as a function of the other primitive terms of S. When c is not definable in S, it is said to be independent of the the other primitive terms.

There is a method, roughly introduced by Alessandro Padoa [21] and further developed by other logicians, which can be employed to show either the independence or the dependence of concepts with respect to the remaining concepts (primitive and previously defined). In fact, Padoa's method gives a necessary and sufficient condition for independence in many formal situations.

In order to present Padoa's method, some preliminary remarks are necessary. Loosely speaking, if we are working in set theory, as our basic framework, an axiomatic system S is characterized by a species of structures [5]. Actually there is a close relationship between species of structures and Suppes predicates [33]; for details see [5]. On the other hand, if our underlying logic is higher-order logic (type theory), S determines a usual higher-order structure [3]. In the first case, our language is the first order language of set theory, and, in the second, it is the language of some type theory. Tarski showed that Padoa's method is valid in the second case [35], and Beth that it is applicable in the first one [1]. A simplified and sufficiently rigorous formulation of Padoa's method, adapted to our exposition, is described in the next paragraph.

Let S be an axiomatic system whose primitive concepts are c_1, c_2, ..., c_n. One of these concepts, say c_i, is independent (undefinable) from the remaining if and only if there are two models of S in which c_1, ..., c_{i-1}, c_{i+1}, ..., c_n have the same interpretation, but the interpretations of c_i in such models are different.

Of course a model of S is a set-theoretical structure in which all axioms of S are true, according to the interpretation of its primitive terms [20].

Next we briefly discuss a well known example of a physical theory stated by means of axioms in order to discuss some points that deserve more details.

4. MSS System of Classical Particle Mechanics

This section is essentially based on the axiomatization of classical particle mechanics due to P. Suppes [31], which is a variant of the formulation by J. C. C. McKinsey, A. C. Sugar and P. Suppes [19]. We call this McKinsey-Sugar-Suppes system of classical particle mechanics as MSS system. MSS system will be useful in order to allow us a discussion about the definability of physical concepts like mass and time and even closed systems.

The reader should not understand that MSS system does faithfully translate all the ideas behind Newtonian mechanics; but it translates, in an intuitive manner, some of the main aspects of Newton's ideas concerning mechanics.

MSS system has six primitive notions: P, T, m, $\mathbf{s}$, $\mathbf{f}$, and $\mathbf{g}$. P and T are sets, m is a real-valued unary function defined on P, $\mathbf{s}$ and $\mathbf{g}$ are vector-valued functions defined on the Cartesian product $P \times T$, and $\mathbf{f}$ is a vector-valued function defined on the Cartesian product $P \times P \times T$. Intuitively, P corresponds to the set of particles and T is to be physically interpreted as a set of real numbers measuring elapsed times (in terms of some unit of time, and measured from some origin of time). $m(p)$ is to be interpreted as the numerical value of the mass of $p \in P$. $\mathbf{s}_p(t)$, where $t \in T$, is a 3-dimensional vector which is to be physically interpreted as the position of particle p at instant t. $\mathbf{f}(p, q, t)$, where p, $q \in P$, corresponds to the internal force that particle q exerts over p, at instant t. And finally, the function $\mathbf{g}(p, t)$ is to be understood as the external force acting on particle p at instant t.

Now, we can give the axioms for MSS system.

Definition 1 $\mathcal{P} = \langle P, T, \mathbf{s}, m, \mathbf{f}, \mathbf{g} \rangle$ *is a MSS system if and only if the following axioms are satisfied:*

P1 *P is a non-empty, finite set.*

P2 *T is an interval of real numbers.*

P3 *If $p \in P$ and $t \in T$, then $\mathbf{s}_p(t)$ is a 3-dimensional vector ($\mathbf{s}_p(t) \in \Re^3$) such that $\frac{d^2\mathbf{s}_p(t)}{dt^2}$ exists.*

P4 *If $p \in P$, then $m(p)$ is a positive real number.*

P5 *If $p, q \in P$ and $t \in T$, then $\mathbf{f}(p, q, t) = -\mathbf{f}(q, p, t)$.*

P6 *If $p, q \in P$ and $t \in T$, then $[\mathbf{s}_p(t), \mathbf{f}(p, q, t)] = -[\mathbf{s}_q(t), \mathbf{f}(q, p, t)]$.*

P7 *If $p, q \in P$ and $t \in T$, then $m(p)\frac{d^2\mathbf{s}_p(t)}{dt^2} = \sum_{q \in P} \mathbf{f}(p, q, t) + \mathbf{g}(p, t)$.*

The brackets [,] in axiom **P6** denote the external product.

Axiom **P5** corresponds to a weak version of Newton's Third Law: to every force there is always a counter-force. Axioms **P6** and **P5**, correspond to the strong version of Newton's Third Law. Axiom **P6** establishes that the direction of force and counter-force is the direction of the line defined by the coordinates of particles p and q.

Axiom **P7** corresponds to Newton's Second Law.

Definition 2 *Let $\mathcal{P} = \langle P, T, \mathbf{s}, m, \mathbf{f}, \mathbf{g}\rangle$ be a MSS system, let P' be a non-empty subset of P, let $\mathbf{s}'$ and m' be, respectively, the restrictions of functions $\mathbf{s}$, and m with their first arguments restricted to P', and let $\mathbf{f}'$ be the restriction of $\mathbf{f}$ with its first two arguments restricted to P'. Then $\mathcal{P}' = \langle P', T, \mathbf{s}', m', \mathbf{f}', \mathbf{g}'\rangle$ is a subsystem of $\mathcal{P}$ iff $\mathbf{g}' : P' \times T \to \Re^3$ is defined as follows:*

$$\mathbf{g}'(p,t) = \sum_{q \in P - P'} \mathbf{f}(p,q,t) + \mathbf{g}(p,t). \tag{2}$$

Theorem 1 *Every subsystem of a MSS system is again a MSS system.*

Definition 3 *Two MSS systems*

$$\mathcal{P} = \langle P, T, \mathbf{s}, m, \mathbf{f}, \mathbf{g}\rangle$$

and

$$\mathcal{P}' = \langle P', T', \mathbf{s}', m', \mathbf{f}', \mathbf{g}'\rangle$$

are equivalent if and only if $P = P'$, $T = T'$, $\mathbf{s} = \mathbf{s}'$, and $m = m'$.

Definition 4 *A MSS system is isolated if and only if for every $p \in P$ and $t \in T$, $\mathbf{g}(p,t) = \langle 0,0,0\rangle$.*

This is a well known definition for isolated systems, which may be termed closed systems as well. It does not say that a closed system does not engage in any causal interaction. Actually, causal interactions (which we interpret as the external forces $\mathbf{g}(p,t)$ and internal forces $\mathbf{f}(p,q,t)$ for all $p, q \in P$ and all $t \in T$) are unavoidable. Besides, this definition can be easily adapted to isolated particles. But the question is: is this really a definition?

The embedding theorem is the following:

Theorem 2 *Every MSS system is equivalent to a subsystem of an isolated MSS system.*

The next theorem can easily be proved by Padoa's method:

Theorem 3 *Mass and internal force are each independent of the remaining primitive notions of MSS system.*

According to Suppes [31]:

> Some authors have proposed that we convert the second law [of Newton], that is, **P7**, into a definition of the total force acting on a particle. [...] It prohibits within the axiomatic framework any analysis of the internal and external forces acting on a particle. That is, if all notions of force are eliminated as primitive and **P7** is used as a definition, then the notions of internal and external force are not definable within the given axiomatic framework.

In [7,8] the authors prove that time is definable (thus dispensable) in some very natural axiomatic frameworks for classical particle mechanics and even thermodynamics. Besides, they prove in the first paper that spacetime is also eliminable in general relativity, classical electromagnetism, Hamiltonian mechanics, classical gauge theories, and Dirac's electron.

Here is one of the theorems proved in the cited papers:

Theorem 4 *Time is eliminable in MSS system.*

The proof is quite trivial. According to Padoa's principle, the primitive concept T in MSS system is independent from the remaining primitive concepts (mass, position, internal force, and external force) iff there are two models of MSS system such that T has two interpretations and the remaining primitive symbols have the same interpretation. But these two interpretations are not possible, since position $\mathbf{s}$, internal force $\mathbf{f}$, and external force $\mathbf{g}$ are functions whose domains depend on T. If we change the interpretation of T, then we change the interpretation of three other primitive concepts, namely, $\mathbf{s}$, $\mathbf{f}$, and $\mathbf{g}$. So, time is not independent and hence can be defined. Since time is definable, it is eliminable.

In [8] the authors show that time is dispensable in thermodynamics as well, at least in a specific (although very natural) axiomatic framework for thermodynamics. Besides, in the same paper they show how to define time and how to rephrase thermodynamics without any explicit reference to time. In the case of MSS system, time can be defined by means of the domain of the functions $\mathbf{s}$, $\mathbf{f}$, and $\mathbf{g}$. A similar procedure is used in [8].

The definition of time in MSS system is an example of a Tarskian definition, since we are defining a set in a set-theoretical species of structures which is known here as MSS system. Nevertheless, the definition of closed system given above is a quite different kind of definition. If we do not make all clear, we can never guess what kind of definition is that one.

The question is: what kind of definition is Definition (4)? It is important to remark that Definition (4) does not refer to a set defined by means of primitive concepts of MSS system. Definition (4) looks like an informal definition by postulates. If that is the case, we cannot use Padoa's method in order to know if that is really a definition. If Padoa's method is applicable, this is a good criterion to see if a given concept is definable by means of other concepts. But Definition (4) seems to be out of our scope. So, further analysis are demanded in future works.

There are other philosophical issues concerning MSS system and definitions, like the anthropomorphical aspect of the concept of force, from Heinrich Hertz's [13] point of view [24,28], and Ernst Mach's [17] principle of inertia [27]. In the case of Hertz's mechanics, the German physicist wanted to define force. And in [24] the author shows that that is a very difficult task. In the case of Mach's mechanics, the Austrian physicist wanted to give a precise philosophical account to the concept of inertia.

5. Other Physical Theories

According to Choi [4], and we agree with that, "it is difficult to find a general formulation of a conservation law of an arbitrary physical quantity in the literature of physics." We agree with the idea that this lack of information makes general statements concerning physical systems very difficult. Nevertheless, we would like to point out that there are some proposals of general frameworks of physical theories in the literature.

In [6] there is a unified treatment for physical theories that allows us to deal with Hamiltonian mechanics, classical gauge field theories, Maxwell electromagnetism, Dirac's electron, and Einstein's general relativity. This unified treatment is given by the next axiomatic system:

Definition 5 *The species of structures of a* classical physical theory *is given by the 9-tuple*

$$\Sigma = \langle M, G, P, \mathcal{F}, \mathcal{A}, \mathcal{I}, \mathcal{G}, B, \bigtriangledown\varphi = \iota\rangle$$

where

1. *M is a finite-dimensional smooth real manifold endowed with a Riemannian metric (associated to spacetime) and G is a finite-dimensional Lie group (associated to transformations among coordinate systems).*

2. *P is a principal fiber bundle $P(M, G)$ over M with Lie group G.*

3. *$\mathcal{F}$, $\mathcal{A}$, and $\mathcal{I}$ are cross-sections of bundles associated to $P(M, G)$, which correspond, respectively, to the field space, potential space, and current or source space.*

4. *$\mathcal{G} \subseteq Diff(M) \otimes \mathcal{G}'$ is the symmetry group of diffeomorphisms of M and $\mathcal{G}'$ is the group of gauge transformations of the principal fiber bundle $P(M, G)$.*

5. *$\bigtriangledown\varphi = \iota$ is a Dirac-like equation, where $\varphi \in \mathcal{F}$ is a field associated to its potential by means of a field equation and $\iota \in \mathcal{I}$ is a current. Such a differential equation is subject to the boundary or initial conditions B.*

We intend, in a future work, to develop some ideas concerning closed systems for general physical systems based on some sort of axiomatic framework like this one. The present paper should be seen as an introduction of a study on a general theory of definition in physical theories.

6. Final Remarks

There are many kinds of definitions in science and in physics. If we want to talk about the definability of concepts, we should make clear what kind of definitions we are talking about. Since there is no general theory of definitions in the literature, we should be very careful about statements like "this is circular" or "this is well-defined". As we discussed above, a definition of an usual concept like that of closed system may raise serious problems from a methodological point of view. Besides, from the logical point of view there are many open problems regarding definitions. For example,

1. Ostensive [22] and operational [2] definitions are widely used in physics. Is there any relationship between these definitions and formal definitions? If we consider that a physical theory is an ordered triple $\langle M, \Delta, \rho \rangle$ where M is a mathematical structure, Δ is a domain of application (the domain of experiments), and ρ is a set of rules connecting M with Δ, then this is a very important question to be answered.

2. How many definitions may exist in a given theory?

3. If mass is a definable concept in some formulations for classical mechanics [29], time is definable in other formulations [7], and even force is dispensable in some axiomatic frameworks [28], then what are the essential concepts in classical mechanics?

4. If time is definable (hence dispensable) in classical mechanics [7] how can we distinguish autonomous from non-autonomous systems?

We are already working on some of these questions.

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References

1. Beth, E. W., 'On Padoa's method in the theory of definition' *Indag. Math.* **15** 330-339 (1953).

2. Bridgman, P. W., *The Logic of Modern Physics* (MacMillan, NY, 1927).

3. Carnap, R., *Introduction to Symbolic Logic and its Applications* (Dover, New York, 1958).

4. Choi, S., 'The conserved quantity theory of causation and closed systems', *Philosophy of Science* **70** 510-530 (2003).

5. da Costa, N. C. A. and R. Chuaqui, 'On Suppes' set-theoretical predicates' *Erkenntnis* **29** 95-112 (1988).

6. da Costa, N. C. A. and F. A. Doria, 'Suppes predicates for classical physics' in J. Echeverria et al. (eds.) *The Space of Mathematics* (Berlin, Walter de Gruyer, 1992).

7. da Costa, N. C. A. and A. S. Sant'Anna, 'The mathematical role of time and spacetime in classical physics', *Foundations of Physics Letters* **14** 553-563 (2001).

8. da Costa, N. C. A. and A. S. Sant'Anna, 'Time in thermodynamics', *Foundations of Physics* **32** 1785-1796 (2002).

9. Goldstein, H., *Classical Mechanics* (Reading, MA, Addison-Wesley, 1980).

10. Gorsky, D. P., *Definition* (Moscow, Progress Publishers, 1981).

11. Gupta, A. and A. Chapuis, *Circularity, Definition, and Truth* (Indian Council of Philosophical Research, 2000)

12. Hempel, C., *Philosophy of Natural Science* (Englewood Cliffs, N.J., Prentice-Hall, 1966).

13. Hertz, H. R., *The Principles of Mechanics*, English translation by D. E. Jones and J. T. Walley (Dover, New York, 1956).

14. Hitchcock, C. R., 'Discussion: Salmon on explanatory relevance', *Philosophy of Science* **62** 304-320 (1995).

15. Jammer, M., *Concepts of mass in contemporary physics and philosophy* (Princeton, Princeton University Press, 2000).

16. Just, W. and Weese, M., *Discovering Modern Set Theory. I.* (Providence, AMS, 1995).

17. Mach, E., *The Science of Mechanics*, English translation by T. J. McCormack (Open Court, New York, 1974).

18. Marion, J. B., *Classical Dynamics of Particles and Systems* (Academic Press, New York, 1970).

19. McKinsey, J. C. C., A. C. Sugar, and P. Suppes (1953), 'Axiomatic foundations of classical particle mechanics', *Journal of Rational Mechanics and Analysis*, **2** 253-272.

20. Mendelson, E., *Introduction to Mathematical Logic* (Chapman & Hall, London, 1997).

21. Padoa, A., 'Essai d'une théorie algébrique des numbres entiers, précédé d'une introduction logique à une théorie déductive quelconque' *Bibliotèque du Congrès International de Philosophie* **3** 309–365 (1900).

22. Pap, A., 'Theory of definition', *Philosophy of Science* **31** 49-54 (1964).

23. Runes, D. D., *Dictionary of Philosophy* (Littlefield, Adams & Co., 1971).

24. Sant'Anna, A. S., 'An axiomatic framework for classical particle mechanics without force', *Philosophia Naturalis* **33** 187-203 (1996).

25. Sant'Anna, A. S., 'Some problems concerning definitions in mathematics and physics', *Boletim da Sociedade Paranaense de Matemática* **20** 21–28 (2002).

26. Sant'Anna, A. S., *O que é Uma Definição* (Barueri, Manole, 2005).

27. Sant'Anna, A. S. and C. A. S. Maia, 'Axioms for Mach's mechanics', *Foundations of Physics Letters* **14** 247-262 (2001).

28. Sant'Anna, A. S. and C. Garcia, 'Gravitation in Hertz's mechanics', *Foundations of Physics Letters*, **16** 559-572 (2003).

29. Schmidt, H.-J., 'A definition of mass in Newton-Lagrange mechanics' *Philsophia Naturalis* **30** 189-207 (1993)

30. McKinsey, J. C. C., A. C. Sugar and P. Suppes, 'Axiomatic foundations of classical particle mechanics', *Journal of Rational Mechanics and Analysis*, **2** 253-272 (1953).

31. Suppes, P., *Introduction to Logic* (Van Nostrand, Princeton, 1957).

32. Suppes, P., 'Philosophy and the sciences', in W. Sieg *Acting and Reflecting* 3-30 (Kluwer Academic, Dordrecht, 1990).

33. Suppes, P., *Representation and Invariance of Scientific Structures* (CSLI, Stanford, 2002).

34. Tarski, A. "On definable sets of real numbers". In: A. Tarski, *Logic, semantics, metamathematics*, 110-142 (Indianapolis, Hacket Pub. Co., 1983).

35. Tarski, A., 'Some methodological investigations on the definability of concepts'. In: A. Tarski, *Logic, semantics, metamathematics* 296–319 (Indianapolis, Hacket Pub. Co., 1983).

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