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Slightly γ-Continuous Functions

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ABSTRACT: The purpose of this paper is to give a new weak form of some types of continuity generalizing strongly α-irresoluteness, α-irresoluteness, α-continuity, precontinuity, semi-continuity, γ-continuity and slightly continuity. In this paper, slightly γ-continuity is introduced and studied. Furthermore, basic properties and preservation theorems of slightly γ-continuous functions are investigated and relationships between slightly γ-continuous functions and graphs are investigated.

Key words: clopen, γ-open, γ-continuity, slightly continuity, slightly γ-continuity.

Contents

1. Introduction and Preliminaries

Functions and of course continuous functions stand among the most important and most researched points in the whole of the Mathematical Science. Many different forms of continuous functions have been introduced over the years. Some of them are strongly α-irresoluteness [9], α-irresoluteness [13], α-continuity [14,15], precontinuity [2,13], semi-continuity [11], γ-continuity [7] and slightly continuity [10,17]. Various interesting problems arise when one considers continuity. Its importance is significant in various areas of mathematics and related sciences.

The aim of this paper is to give a new weaker form of some types of continuity including strongly α-irresoluteness, α-irresoluteness, α-continuity, precontinuity, semi-continuity, γ-continuity and slightly continuity. In this paper, slightly γ-continuity is introduced and studied. Moreover, basic properties and preservation theorems of slightly γ-continuous functions are investigated and relationships between slightly γ-continuous functions and graphs are investigated.

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In Section 2, the notion of slightly γ-continuous functions is introduced and characterizations and some relationships of γ-continuous functions and basic properties of slightly γ-continuous functions are investigated and obtained. The relationships between slightly γ-continuity and connectedness are investigated. In Section 3 and in Section 4, the relationships between slightly γ-continuity and compactness and the relationships between slightly γ-continuity and separation axioms and graphs are obtained. In Section 5, the relationships slightly γ-continuity and the other types of continuity are investigated.

Throughout the present paper, X and Y are always topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by $int(A)$ and $cl(A)$, repectively.

A subset A of a space X is said to be preopen [13] (resp. semi-open [11], α-open [15], b-open [1] or γ-open [7] or sp-open [6]) if $A \subset int(cl(A))$ (resp. $A \subset cl(int(A))$, $A \subset int(cl(int(A)))$, $A \subset cl(int(A)) \cup int(cl(A))$).

The complement of a γ-open set is said to be γ-closed [7]. The intersection of all γ-closed sets of X containing A is called the γ-closure [7] of A and is denoted by $\gamma cl(A)$. The union of all γ-open sets of X contained A is called γ-interior of A and is denoted by $\gamma int(A)$.

The family of all α-open (resp. γ-open, γ-closed, clopen, γ-clopen) sets of X is denoted by $\alpha O(X)$ (resp. $\gamma O(X)$, $\gamma C(X)$, $CO(X)$, $\gamma CO(X)$).

Definition 1 *A function $f : X \to Y$ is γ-continuous [7] if $f^{-1}(V)$ is γ-open set in X for each open set V of Y.*

Definition 2 *A function $f : X \to Y$ is slightly continuous [10] if $f^{-1}(V)$ is open set in X for each clopen set V of Y.*

2. Slightly γ-continuous functions

In this section, the notion of slightly γ-continuous functions is introduced and characterizations and some relationships of γ-continuous functions and basic properties of slightly γ-continuous functions are investigated and obtained.

Definition 3 *A function $f : X \to Y$ is called:*
(1) slightly γ-continuous at a point $x \in X$ if for each clopen subset V in Y containing $f(x)$, there exists a γ-open subset U in X containing x such that $f(U) \subset V$.
(2) slightly γ-continuous if it has this property at each point of X.

Theorem 2.1 *Let (X, τ) and (Y, υ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:*
(1) f is slightly γ-continuous;
(2) for every clopen set $V \subset Y$, $f^{-1}(V)$ is γ-open;
(3) for every clopen set $V \subset Y$, $f^{-1}(V)$ is γ-closed;
(4) for every clopen set $V \subset Y$, $f^{-1}(V)$ is γ-clopen.

Proof. (1) $\Rightarrow$ (2) : Let V be a clopen subset of Y and let $x \in f^{-1}(V)$. Since $f(x) \in V$, by (1), there exists a γ-open set U_x in X containing x such that $U_x \subset f^{-1}(V)$. We obtain that $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Thus, $f^{-1}(V)$ is γ-open.

(2) $\Rightarrow$ (3) : Let V be a clopen subset of Y. Then, $Y \backslash V$ is clopen. By (2), $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is γ-open. Thus, $f^{-1}(V)$ is γ-closed.

(3) $\Rightarrow$ (4) : It can be shown easily.

(4) $\Rightarrow$ (1) : Let V be a clopen subset in Y containing $f(x)$. By (4), $f^{-1}(V)$ is γ-clopen. Take $U = f^{-1}(V)$. Then, $f(U) \subset V$. Hence, f is slightly γ-continuous.

Lemma 2.2 *Let A and X_0 be subsets of a space (X, τ). If $A \in \gamma O(X)$ and $X_0 \in \alpha O(X)$, then $A \cap X_0 \in \gamma O(X_0)$ [1,7].*

Theorem 2.3 *If $f : X \to Y$ is slightly γ-continuous and $A \in \alpha O(X)$, then the restriction $f\mid_A : A \to Y$ is slightly γ-continuous.*

Proof. Let V be a clopen subset of Y. We have $(f\mid_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is γ-open and A is α-open, it follows from the previous lemma that $(f\mid_A)^{-1}(V)$ is γ-open in the relative topology of A. Thus, $f\mid_A$ is slightly α-continuous.

Lemma 2.4 *Let $A \subset X_0 \subset X$, $A \in \gamma O(X_0)$ and $X_0 \in \alpha O(X)$, then $A \in \gamma O(X)$ [7].*

Theorem 2.5 *Let $f : X \to Y$ be a function and $\Sigma = \{U_i : i \in I\}$ be a cover of X such that $U_i \in \alpha O(X)$ for each $i \in I$. If $f\mid_{U_i}$ is slightly γ-continuous for each $i \in I$, then f is a slightly γ-continuous function.*

Proof. Suppose that V is any clopen set of Y. Since $f\mid_{U_i}$ is slightly γ-continuous for each $i \in I$, it follows that $(f\mid_{U_i})^{-1}(V) \in \gamma(U_i)$. We have

$f^{-1}(V) = \bigcup_{i \in I}(f^{-1}(V) \cap U_i) = \bigcup_{i \in I}(f\mid_{U_i})^{-1}(V)$. Then, Lemma 2.4 we obtain $f^{-1}(V) \in \gamma O(X)$ which means that f is slightly γ-continuous.

Theorem 2.6 *Let $f : X \to Y$ be a function and $x \in X$. If there exists $U \in \alpha O(X)$ such that $x \in U$ and the restriction of f to U is a slightly γ-continuous function at x, then f is slightly γ-continuous at x.*

Proof. Suppose that $F \in CO(Y)$ containing $f(x)$. Since $f\mid_U$ is slightly γ-continuous at x, there exists $V \in \gamma O(U)$ containing x such that $f(V) = (f\mid_U)(V) \subset F$. Since $U \in \alpha O(X)$ containing x, it follows from Lemma 2.4 that $V \in \gamma O(X)$ containing x. This shows clearly that f is slightly γ-continuous at x.

Theorem 2.7 *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. Then g is slightly γ-continuous if and only if f is slightly γ-continuous.*

Proof. Let $V \in CO(Y)$, then $X \times V \in CO(X \times Y)$. Since g is slightly γ-continuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \gamma O(X)$. Thus, f is slightly γ-continuous.

Conversely, let $x \in X$ and let W be a closed subset of $X \times Y$ containing $g(x)$.. Then $W \cap (\{x\} \times Y)$ is clopen in $\{x\} \times Y$ containing $g(x)$.. Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y/(x,y) \in W\}$ is a clopen subset of Y. Since f is slightly γ-continuous , $\bigcup\{f^{-1}(y)/(x,y) \in W\}$ is a γ-open subset of X. Further $x \in \bigcup\{f^{-1}(y)/(x,y) \in W\} \subset g^{-1}(W)$. Hence $g^{-1}(W)$ is γ-open . Then g is slightly γ-continuous.

Definition 4 *A function $f : X \to Y$ is called:*
(i) f γ-irresolute if for every γ-open subset G of Y, $f^{-1}(G)$ is γ-open in Y.
(ii) f γ-open if for every γ-open subset A of X, $f(A)$ is γ-open in Y.

Theorem 2.8 *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*
(1) If f is γ-irresolute and g is slightly γ-continuous, then $g \circ f : X \to Z$ is slightly γ-continuous.
(2) If f is γ-irresolute and g is γ-continuous, then $g \circ f : X \to Z$ is slightly γ-continuous.
(3) If f is γ-irresolute and g is slightly continuous, then $g \circ f : X \to Z$ is slightly γ-continuous.

Proof. (1) Let V be any clopen set in Z. Since g is slightly γ-continuous, $g^{-1}(V)$ is γ-open. Since f is γ-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is γ-open. Therefore, $g \circ f$ is slightly γ-continuous.

(2) and (3) can be obtained smilarly.

Theorem 2.9 *Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is γ-open and surjective and $g \circ f : X \to Z$ is slightly γ-continuous, then g is slightly γ-continuous.*

Proof. Let V be any clopen set in Z. Since $g \circ f$ is slightly γ-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is γ-open. Since f is γ-open, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is γ-open. Hence, g is slightly γ-continuous.

Combining the previous two theorem, we obtain the following result.

Theorem 2.10 *Let $f : X \to Y$ be surjective, γ-irresolute and γ-open and $g : Y \to Z$ be a function. Then $g \circ f : X \to Z$ is slightly γ-continuous if and only if g is slightly γ-continuous.*

Definition 5 *(i) A filter base Λ is said to be γ-convergent to a point x in X if for any $U \in \gamma O(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.*
(ii) A filter base Λ is said to be co-convergent to a point x in X if for any $U \in CO(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.

Theorem 2.11 *If a function $f : X \to Y$ is slightly γ-continuous, then for each point $x \in X$ and each filter base Λ in X γ-converging to x, the filter base $f(\Lambda)$ is co-convergent to $f(x)$.*

Proof. Let $x \in X$ and Λ be any filter base in X γ-converging to x. Since f is slightly γ-continuous, then for any $V \in CO(Y)$ containing $f(x)$, there exists a $U \in \gamma O(X)$ containing x such that $f(U) \subset V$. Since Λ is γ-converging to x, there exists a $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Lambda)$ is co-convergent to $f(x)$.

Recall that, a space X is called γ-connected [7] provided that X is not the union of two disjoint nonempty γ-open sets.

Theorem 2.12 *If $f : X \to Y$ is slightly γ-continuous surjective function and X is γ-connected space, then Y is connected space.*

Proof. Suppose that Y is not connected space. Then there exists nonempty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since f is slightly γ-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are γ-closed and γ-open in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not γ-connected. This is a contradiction. Hence, Y is connected.

Definition 6 *A topological space X is called hyperconnected [19] if every nonempty open subset of X is dense in X. It is well-known that every hyperconnected space is connected but not conversely.*

Remark 2.13 *The following example shows that slightly γ-continuous surjection do not necessarily preserve hyperconnectedness.*

Example 2.14 *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly γ-continuous surjective. (X, τ) is hyperconnected. But (X, σ) is not hyperconnected.*

3. Covering properties

In this section, the relationships between slightly γ-continuous functions and compactness are investigated.

Definition 7 *A space X is said to be mildly compact [18] (respectively γ-compact [7]) if every clopen cover (resp. γ-open cover) of X has a finite subcover.*

A subset A of a space X is said to be mildly compact (respectively γ-compact) relative to X if every cover of A by clopen (resp. γ-open) sets of X has a finite subcover.

A subset A of a space X is said to be mildly compact (respectively γ-compact) if the subspace A is mildly compact (resp. γ-compact).

Theorem 3.1 *If a function $f : X \to Y$ is slightly γ-continuous and K is γ-compact relative to X, then $f(K)$ is mildly compact in Y.*

Proof. Let $\{H_\alpha : \alpha \in I\}$ be any cover of $f(K)$ by clopen sets of the subspace $f(K)$. For each $\alpha \in I$, there exists a clopen set K_α of Y such that $H_\alpha = K_\alpha \cap f(K)$.

For each $x \in K$, there exists $\alpha_x \in I$ such that $f(x) \in K_{\alpha_x}$ and there exists $U_x \in \gamma O(X)$ containing x such that $f(U_x) \subset K_{\alpha_x}$. Since the family $\{U_x : x \in K\}$ is a cover of K by γ-open sets of K, there exists a finete subset K_0 of K such that $K \subset \{U_x : x \in K_0\}$. Therefore, we obtain $f(K) \subset \bigcup\{f(U_x) : x \in K_0\}$ which is a subset of $\bigcup\{K_{\alpha_x} : x \in K_0\}$. Thus $f(K) = \bigcup\{H_{\alpha_x} : x \in K_0\}$ and hence $f(K)$ is mildly compact.

Corollary 3.2 *If $f : X \to Y$ is slightly γ-continuous surjection and X is γ-compact, then Y is mildly compact.*

Definition 8 *A space X said to be:*
(1) mildly countably compact [18] if every clopen countably cover of X has a finite subcover.
(2) mildly Lindelof [18] if every cover of X by clopen sets has a countable subcover.
(3) countably γ-compact if every γ-open countably cover of X has a finite subcover.
(4) γ-Lindelof if every γ-open cover of X has a countable subcover.
(5) γ-closed-compact if every γ-closed cover of X has a finite subcover.
(6) countably γ-closed-compact if every countable cover of X by γ-closed sets has a finete subcover.
(7) γ-closed-Lindelof if every cover of X by γ-closed sets has a countable subcover.

Theorem 3.3 *Let $f : X \to Y$ be a slightly γ-continuous surjection. Then the following statements hold:*
(1) if X is γ-Lindelof, then Y is mildly Lindelof.
(2) if X is countably γ-compact, then Y is mildly countably compact.

Proof. We prove (1), the proof of (2) being entirely analogous.

Let $\{V_\alpha : \alpha \in I\}$ be any clopen cover of Y. Since f is slightly γ-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a γ-open cover of X. Since X is γ-Lindelof, there exists a countable subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is mildly Lindelof.

Theorem 3.4 *Let $f : X \to Y$ be a slightly γ-continuous surjection. Then the following statements hold:*
(1) if X is γ-closed-compact, then Y is mildly compact.
(2) if X is γ-closed-Lindelof, then Y is mildly Lindelof.
(3) if X is countably γ-closed-compact, then Y is mildly countably compact.

Proof. It can be obtained smilarly as Theorem 3.3.

4. Separation axioms

In this section, the relationships between slightly γ-continuous functions and separation axioms are investigated.

Definition 9 *A space X is said to be:*
(i) γ-T_1 [4,5] if for each pair of distinct points x and y of X, there exist γ-open

sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.
(ii) A space X is said to be γ-T_2 (γ-Hausdorff) [4,5] if for each pair of distinct points x and y in X, there exist disjoint γ-open sets U and V in X such that $x \in U$ and $y \in V$.
(iii) A space X is said to be clopen T_1 if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.
(iv) A space X is said to be clopen T_2 (clopen Hausdorff or ultra-Hausdorff) [18] if for each pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X such that $x \in U$ and $y \in V$.

Remark 4.1 *[4,5]*
(i) A topological space (X,τ) is γ-T_1 if and only if the singletons are γ-closed sets.
(ii) A topological space (X,τ) is γ-T_2 if and only if the intersection of all γ-closed γ-neighourhoods of each point of X is reduced to that point.

Remark 4.2 *The following implications are hold for a topological space X:*
(1) clopen $T_1 \Rightarrow T_1$,
(2) $T_1 \Rightarrow \gamma$-T_1.

None of these implications is reversible.

Example 4.3 *Let R be the real numbers with the finite complements topology τ. Then (R,τ) is T_1 but not clopen T_1.*

Example 4.4 *Let $X = \{a,b,c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b,c\}\}$. Then (X,τ) is γ-T_1 but not T_1.*

Theorem 4.5 *If $f : X \to Y$ is a slightly γ-continuous injection and Y is clopen T_1, then X is γ-T_1.*

Proof. Suppose that Y is clopen T_1. For any distinct points x and y in X, there exist V, $W \in CO(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is slightly γ-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are γ-open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is γ-T_1.

Theorem 4.6 *If $f : X \to Y$ is a slightly γ-continuous injection and Y is clopen T_2, then X is γ-T_2.*

Proof. For any pair of distinct points x and y in X, there exist disjoint clopen sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is slightly γ-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are γ-open in X containing x and y respectively. Therefore $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ because $U \cap V = \emptyset$. This shows that X is γ-T_2.

Lemma 4.7 *The intersection of an open and a γ-open set is a γ-open set [1].*

Theorem 4.8 *If $f : X \to Y$ is slightly continuous function and $g : X \to Y$ is slightly γ-continuous function and Y is clopen Hausdorff, then $E = \{x \in X : f(x) = g(x)\}$ is γ-closed in X.*

Proof. If $x \in X \backslash E$, then it follows that $f(x) \neq g(x)$. Since Y is clopen Hausdorff, there exist $f(x) \in V \in CO(Y)$ and $g(x) \in W \in CO(Y)$ such that $V \cap W = \emptyset$. Since f is slightly continuous and g is slightly γ-continuous, then $f^{-1}(V)$ is open and $g^{-1}(W)$ is γ-open in X with $x \in f^{-1}(V)$ and $x \in g^{-1}(W)$. Set $O = f^{-1}(V) \cap g^{-1}(W)$. By Lemma 4.7, O is γ-open. Therefore $f(O) \cap g(O) = \emptyset$ and it follows that $x \notin \gamma cl(E)$. This shows that E is γ-closed in X.

Definition 10 *A space is called clopen regular (respectively γ-regular) if for each clopen (respectively γ-closed) set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.*

Definition 11 *A space is said to be clopen normal (respectively γ-normal) if for every pair of disjoint clopen (respectively γ-closed) subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.*

Theorem 4.9 *If f is slightly γ-continuous injective open function from a γ-regular space X onto a space Y, then Y is clopen regular.*

Proof. Let F be clopen set in Y and be $y \notin F$. Take $y = f(x)$. Since f is slightly γ-continuous, $f^{-1}(F)$ is a γ-closed set. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is γ-regular, there exist disjoint open sets U and V such that $G \subset U$ and $x \in V$. We obtain that $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. This shows that Y is clopen regular.

Theorem 4.10 *If f is slightly γ-continuous injective open function from a γ-normal space X onto a space Y, then Y is clopen normal.*

Proof. Let F_1 and F_2 be disjoint clopen subsets of Y. Since f is slightly γ-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are γ-closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \emptyset$. Since X is γ-normal, there exist disjoint open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint open sets. Thus, Y is clopen normal.

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 12 *A graph $G(f)$ of a function $f : X \to Y$ is said to be strongly γ-co-closed if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in \gamma CO(X)$ containing x and $V \in CO(Y)$ containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 4.11 *A graph $G(f)$ of a function $f : X \to Y$ is strongly γ-co-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in \gamma CO(X)$ containing x and $V \in CO(Y)$ containing y such that $f(U) \cap V = \emptyset$.*

Theorem 4.12 *If $f : X \to Y$ is slightly γ-continuous and Y is clopen T_1, then $G(f)$ is strongly γ-co-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$, then $f(x) \neq y$ and there exists a clopen set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is slightly γ-continuous, then $f^{-1}(V) \in \gamma CO(X)$ containing x. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Therefore, we obtain $f(U) \cap (Y\backslash V) = \emptyset$ and $Y\backslash V \in CO(Y)$ containing y. This shows that $G(f)$ is strongly γ-co-closed in $X \times Y$.

Corollary 4.13 *If $f : X \to Y$ is slightly γ-continuous and Y is clopen Hausdorff, then $G(f)$ is strongly γ-co-closed in $X \times Y$.*

Theorem 4.14 *Let $f : X \to Y$ has a strongly γ-co-closed graph $G(f)$. If f is injective, then X is γ-T_1.*

Proof. Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y)\backslash G(f)$. By Lemma 4.11, there exist a γ-clopen set U of X and $V \in CO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \emptyset$. Hence $U \cap f^{-1}(V) = \emptyset$ and $y \notin U$. This implies that X is γ-T_1.

Theorem 4.15 *Let $f : X \to Y$ has a strongly γ-co-closed graph $G(f)$. If f is surjective γ-open function, then Y is γ-T_2.*

Proof. Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y)\backslash G(f)$. By Definition 12, there exist a γ-clopen set U of X and $V \in CO(Y)$ such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \emptyset$. Then, we have $f(U) \cap V = \emptyset$. Since f is γ-open, then $f(U)$ is γ-open such that $f(x) = y_1 \in f(U)$. This implies that Y is γ-T_2.

5. Relationships

Definition 13 *A function $f : X \to Y$ is semi-continuous [11] if $f^{-1}(V)$ is semi-open set in X for each open set V of Y.*

Definition 14 *A function $f : X \to Y$ is called precontinuous [2,13] if $f^{-1}(V)$ is preopen set in X for each open set V of Y.*

Definition 15 *A function $f : X \to Y$ is said to be α-continuous [14] if $f^{-1}(V)$ is α-open in X for every open set V of Y.*

Definition 16 *A function $f : X \to Y$ is called α-irresolute [12] if $f^{-1}(V)$ is α-open set in X for each α-open set V of Y.*

Definition 17 *A function $f : X \to Y$ is said to be strongly α-irresolute [9] if for each $x \in X$ and each α-open subset V of Y containing $f(x)$, there exists a open subset U of X containing x such that $f(U) \subset V$.*

Remark 5.1 *The following diagram holds:*

$$\begin{array}{ccc}
 & & slightlycontinuous \\
 & & \Downarrow \\
precontinuous & \Rightarrow \gamma - continuous & \Rightarrow slightly\gamma - continuous \\
\Uparrow & \Uparrow & \\
\alpha - continuous & \Rightarrow semi - continous & \\
\Uparrow & & \\
\alpha - irresolute & & \\
\Uparrow & & \\
strongly\alpha - irresolute & &
\end{array}$$

None of these implications is reversible.

Example 5.2 *Let R and N be the real numbers and natural numbers, respectively. Take two topologies on R as $\tau = \{R, \emptyset, N\}$ and $\upsilon = \{R, \emptyset, R \backslash N\}$. Let $f : (R, \tau) \to (R, \upsilon)$ be an identity function. Then, f is slightly γ-continuous, but it is not γ-continuous.*

Example 5.3 *Let R be the real numbers. Take two topologies on R as τ_u and τ_D where τ_u is usual topology and τ_D is discrete topology. Let $f : (R, \tau_u) \to (R, \tau_D)$ be an identity function. Then, f is slightly γ-continuous, but it is not slightly continuous.*

The other implications are not reversible as shown in several papers [6,7,9,11, 12,13,14].

Recall that a space is 0-dimensional if its topology has a base consisting of clopen sets.

Theorem 5.4 *If $f : X \to Y$ is slightly γ-continuous and Y is a 0-dimensional space, then f is γ-continuous.*

Proof. Let $x \in X$ and let V be an open subset of Y containing $f(x)$. Since Y is a 0-dimensional, there exists a clopen set U containing $f(x)$ such that $U \subset V$. Since f is slightly γ-continuous, then there exists an γ-open subset G in X containing x such that $f(G) \subset U \subset V$. Thus, f is γ-continuous.

Recall that a space X is said to be:
(1) submaximal [3] if each dense subset of X is open in X,
(2) extremally disconnected [3] if the closure of each open set of X is open in X.

Theorem 5.5 *If (X, τ) a submaximal extremally disconnected space, then the following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*
(1) f is slightly γ-continuous;
(2) f is slightly continuous.

Proof. (1)⇒(2): This follows from the fact that if (X, τ) is a submaximal extremally disconnected space, then $\tau = \gamma O(X)$.

(2)⇒(1): Obvious.

Theorem 5.6 *If X is a submaximal extremally disconnected space and Y is a 0-dimensional space, then the following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*
(1) f is slightly γ-continuous;
(2) f is α-continuous.

Proof. (1)$\Rightarrow$(2): Let $x \in X$ and let V be an open subset of Y containing $f(x)$. Since Y is 0-dimensional, there exists a clopen set U containing $f(x)$ such that $U \subset V$. Since f is slightly γ-continuous, then there exists a γ-open subset G in X containing x such that $f(G) \subset U \subset V$. Since X is a submaximal extremally disconnected space, then $\gamma O(X) = \alpha O(X)$. Hence, f is α-continuous.

(2)$\Rightarrow$(1): Obvious.

Corollary 5.7 *Let Y be a 0-dimensional space and X be a submaximal extremally disconnected space. The following statements are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*
(1) f is slightly γ-continuous;
(2) f is slightly continuous;
(3) f is α-continuous;
(4) f is precontinuous;
(5) f is semi-continuous;
(6) f is γ-continuous.

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