Bol. Soc. Paran. Mat.
(3s.) **v. 21** 1/2 (2003): 1–14.

Characterizations of low separation axioms via α-open sets and α-closure operator

M. Caldas, D. N. Georgiou and S. Jafari

ABSTRACT: In this paper, we introduce and investigate some weak separation axioms by using the notions of α-open sets and the α-closure operator.

Contents

1. Introduction

The notion of α-open set was introduced by O. Njåstad [25] in 1965. Since then it has been widely investigated in the literature (see, [1], [2], [6], [11], [13], [14], [16], [18], [19], [21], [22], [23], [26], [28], [29], [30], [31], [32]). In this paper, we offer some new low separation axioms by utilizing α-open sets and α-closure operator. We also characterize their fundamental properties.

Throughout this paper, by (X, τ) and (Y, σ) (or X and Y) we always mean topological spaces. A subset A of a topological space (X, τ) is called *α-open* [25] (resp. *semi-open* [17] and *preopen* [24]) if $A \subseteq Int(Cl(Int(A)))$ (resp. $A \subseteq Cl(Int(A))$ and $A \subseteq Int(Cl(A))$). The complement of an α-open (resp. semi-open and preopen) set is called *α-closed* (resp. *semi-closed* [9] and *preclosed* [24]). By $\alpha\mathrm{O}(X, \tau)$ (resp. $\alpha\mathrm{C}(X, \tau)$), we denote the family of all α-open (resp. α-closed) sets of X. The intersection of all α-closed (resp. semi-closed and preclosed) sets containing A is called the *α-closure* (resp. *semi-closure* [8] and *preclosure* [27]) of A, denoted by $Cl_\alpha(A)$ (resp. $sCl(A)$ and $pCl(A)$). A subset A is *α-closed* if and only if $A = Cl_\alpha(A)$. A set U in a topological space (X, τ) is an *α-neighborhood* [19] of a point x if U contains an α-open set V such that $x \in V$. Recall that a topological space (X, τ) is said to be:
(i) Weakly-R_0 [10] (resp. weakly semi-R_0 [3] and weakly pre-R_0 [15]) if $\cap_{x\in X} Cl(\{x\}) = \emptyset$ (resp. $\cap_{x\in X} sCl(\{x\}) = \emptyset$ and $\cap_{x\in X} pCl(\{x\}) = \emptyset$).

1991 *Mathematics Subject Classification:* 54B05, 54C08, 54D05

(ii) Semi-R_0 [20] (resp. pre-R_0 [5]) if every semi-open (resp. preopen) set contains the semi-closure (resp. preclosure) of each of its singletons.

Corollary 1.1 *Let A be a subset of a topological space (X,τ). Then $Cl_\alpha(A) = A\bigcup Cl(Int(Cl(A)))$.*

Corollary 1.2 *$Cl_\alpha(A)$ is α-closed , i.e. $Cl_\alpha(Cl_\alpha(A)) = Cl_\alpha(A)$.*

Lemma 1.3 *For subsets A and A_i $(i \in I)$ of a space (X,τ), the following hold:*
(1) $A \subset Cl_\alpha(A)$.
(2) If $A \subset B$, then $Cl_\alpha(A) \subset Cl_\alpha(B)$.
(3) $Cl_\alpha(\cap\{A_i : i \in I\}) \subset \cap\{Cl_\alpha(A_i) : i \in I\}$.
(4) $Cl_\alpha(\cup\{A_i : i \in I\}) = \cup\{Cl_\alpha(A_i) : i \in I\}$.

2. Preliminaries

In this section we recall the definitions of α-T_i, s-D_i and p-D_i spaces, $i = 0, 1, 2$.

A subset A of a topological space (X,τ) is called:
(i) sD-set [7] if there are two semi-open sets U and V such that $U \neq X$ and $A{=}U - V$.
(ii) pD-set [12] if there are two preopen sets U and V such that $U \neq X$ and $A{=}U - V$.
Observe that every semi-open (respectively, preopen) set U different from X is a sD-set (respectively, pD-set) if $A{=}U$ and $V{=}\emptyset$.

A topological space (X,τ) is said to be:
(iii) α-T_0 [21] if for any distinct pair of points in X, there is an α-open set containing one of the points but not the other.
(iv) α-T_1 [21] if for any distinct pair of points x and y in X, there is an α-open U in X containing x but not y and an α-open set V in X containing y but not x.
(v) α-T_2 [18] if for any distinct pair of points x and y in X, there exist α-open sets U and V in X containing x and y, respectively, such that $U \cap V = \emptyset$.
(vi) *s-D_0* [7] if for any distinct pair of points x and y of X there exists a sD-set of X containing x but not y or a sD-set of X containing y but not x.
(vii) *s-D_1* [7] if for any distinct pair of points x and y of X there exists a sD-set of X containing x but not y and a sD-set of X containing y but not x.
(viii) *s-D_2* [7] if for any distinct pair of points x and y of X there exists disjoint sD-sets G and E of X containing x and y, respectively.
(ix) *p-D_0* [12] if for any distinct pair of points x and y of X there exists a pD-set of X containing x but not y or a pD-set of X containing y but not x.
(x) *p-D_1* [12] if for any distinct pair of points x and y of X there exists a pD-set of X containing x but not y and a pD-set of X containing y but not x.
(xi) *p-D_2* [12] if for any distinct pair of points x and y of X there exists disjoint pD-sets G and E of X containing x and y, respectively.

Remark 2.1 *(i) If (X,τ) is α-T_i, then it is α-T_{i-1}, $i = 1, 2$.*
(ii) If (X,τ) is s-D_i, then it is s-D_{i-1}, $i = 1, 2$.
(iii) If (X,τ) is p-D_i, then it is p-D_{i-1}, $i = 1, 2$.

3. αD-sets and associated separation axioms

Definition 1 *A subset A of a topological space X is called an αD-set if there are two $U, V \in \alpha O(X,\tau)$ such that $U \neq X$ and A=$U - V$.*

Observe that every α-open set U different from X is an αD-set if A=U and V=$\emptyset$.

Example 3.1 *Let (X,τ) be a topological space such that $X = \{a,b,c,d\}$ and $\tau = \{\emptyset, X, \{a,b\}\}$. Clearly, the set $\{a,c,d\}$ is pD-set but it is not αD-set.*

Example 3.2 *Let (X,τ) be a topological space such that $X = \{a,b,c,d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$. Clearly, the set $\{b,c,d\}$ is sD-set but it is not αD-set.*

Definition 2 *A topological space (X,τ) is called α-D_0 if for any distinct pair of points x and y of X there exists an αD-set of X containing x but not y or an αD-set of X containing y but not x.*

Definition 3 *A topological space (X,τ) is called α-D_1 if for any distinct pair of points x and y of X there exists an αD-set of X containing x but not y and an αD-set of X containing y but not x.*

Definition 4 *A topological space (X,τ) is called α-D_2 if for any distinct pair of points x and y of X there exists disjoint αD-sets G and E of X containing x and y, respectively.*

Remark 3.3 *(i) If (X,τ) is α-T_i, then (X,τ) is α-D_i, $i = 0,1,2$.*
(ii) If (X,τ) is α-D_i, then it is α-D_{i-1}, $i = 1,2$.

Theorem 3.4 *For a topological space (X,τ) the following statements hold:*
(1) (X,τ) is α-D_0 if and only if it is α-T_0.
(2) (X,τ) is α-D_1 if and only if it is α-D_2.

Proof. (1) The sufficiency is stated in Remark 3.3(i). To prove necessity, let (X,τ) be α-D_0. Then for each distinct pair $x, y \in X$, at least one of x, y, say x, belongs to an αD-set G where $y \notin G$. Let $G = U_1 \setminus U_2$ such that $U_1 \neq X$ and U_1, $U_2 \in \alpha O(X,\tau)$. Then $x \in U_1$. For $y \notin G$ we have two cases: (a) $y \notin U_1$; (b) $y \in U_1$ and $y \in U_2$.

In case (a), $x \in U_1$ but $y \notin U_1$;

In case (b), $y \in U_2$ but $x \notin U_2$. Hence X is α-T_0.

(2) Sufficiency. Remark 3.3(ii).

Necessity. Suppose that X is α-D_1. Then for each distinct pair $x, y \in X$, we have αD-sets G_1, G_2 such that $x \in G_1$, $y \notin G_1$; $y \in G_2$, $x \notin G_2$. Let G_1= $U_1 \backslash U_2$, $G_2 = U_3 \backslash U_4$. By $x \notin G_2$, it follows that either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$. Now we consider two cases.

(1) $x \notin U_3$. By $y \notin G_1$ we have two subcases:

(a) $y \notin U_1$. By $x \in U_1 \backslash U_2$, it follows that $x \in U_1 \backslash$ $(U_2 \cup U_3)$ and by $y \in U_3 \backslash U_4$ we have $y \in U_3 \backslash (U_1 \cup U_4)$. Hence $(U_1 \backslash (U_2 \cup U_3)) \cap (U_3 \backslash (U_1 \cup U_4) = \emptyset$.

(b) $y \in U_1$ and $y \in U_2$. We have $x \in U_1 \backslash U_2$, $y \in U_2$. $(U_1 \backslash U_2) \cap U_2 = \emptyset$.

(2) $x \in U_3$ and $x \in U_4$. We have $y \in U_3 \backslash U_4$, $x \in U_4$. $(U_3 \backslash U_4) \cap U_4 = \emptyset$. Therefore X is α-D_2.

Theorem 3.5 *If (X, τ) is α-D_1, then it is α-T_0.*

Proof. Remark 3.3 and Theorem 3.4.

Example 3.6 *Let (X, τ) be a topological space such that $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Clearly the space X is α-T_0 but it is not α-D_1.*

Theorem 3.7 *A topological space (X, τ) is α-T_0 if and only if for each pair of distinct points x, y of X, $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$.*

Proof. Sufficiency. Suppose that $x, y \in X$, $x \neq y$ and $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. Let $z \in X$ such that $z \in Cl_\alpha(\{x\})$ but $z \notin Cl_\alpha(\{y\})$. We claim that $x \notin Cl_\alpha(\{y\})$. For, if $x \in Cl_\alpha(\{y\})$ then $Cl_\alpha(\{x\}) \subset Cl_\alpha(\{y\})$. This contradicts the fact that $z \notin Cl_\alpha(\{y\})$. Consequently x belongs to the α-open set $[Cl_\alpha(\{y\})]^c$ to which y does not belong.

Necessity. Let (X, τ) be an α-T_0 space and x, y be any two distinct points of X. There exists an α-open set G containing x or y, say x but not y. Then G^c is an α-closed set which $x \notin G^c$ and $y \in G^c$. Since $Cl_\alpha(\{y\})$ is the smallest α-closed set containing y (Corollary 1.1), $Cl_\alpha(\{y\}) \subset G^c$, and therefore $x \notin Cl_\alpha(\{y\})$. Hence $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$.

Theorem 3.8 *A topological space (X, τ) is α-T_1 if and only if the singletons are α-closed sets.*

Proof. Let (X, τ) be α-T_1 and x any point of X. Suppose $y \in \{x\}^c$. Then $x \neq y$ and so there exists an α-open set U_y such that $y \in U_y$ but x$\notin U_y$. Consequently $y \in U_y \subset \{x\}^c$ i.e., $\{x\}^c = \bigcup\{U_y / y \in \{x\}^c\}$ which is α-open.

Conversely, suppose $\{p\}$ is α-closed for every $p \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in \{x\}^c$. Hence$\{x\}^c$ is an α-open set containing y but not x. Similarly $\{y\}^c$ is an α-open set containing x but not y. Accordingly X is an α-T_1 space.

Definition 5 *A point $x \in X$ which has X as an α-neighborhood is called an α*-neat point.

Theorem 3.9 *For an α-T_0 topological space (X, τ) the following are equivalent:*
(1) (X, τ) is α-D_1;
(2) (X, τ) has no α-neat point.

Proof. (1) $\rightarrow$ (2). Since (X, τ) is α-D_1, then each point x of X is contained in an αD-set *O*=*U* $-$ *V* and thus in *U*. By definition $U \neq X$. This implies that x is not an α-neat point.

(2) $\rightarrow$ (1). If X is α-T_0, then for each distinct pair of points $x, y \in X$, at least one of them, x(say) has an α-neighborhood U containing x and not y. Thus U which is different from X is an αD-set. If X has no α-neat point, then y is not an α-neat point. This means that there exists an α-neighborhood V of y such that $V \neq X$. Thus $y \in (V - U)$ but not x and $V - U$ is an αD-set. Hence X is α-D_1.

Remark 3.10 *It is clear that an α-T_0 topological space (X, τ) is not α-D_1 if and only if there is a unique α-neat point in X. It is unique because if x and y are both α-neat point in X, then at least one of them say x has an α-neighborhood U containing x but not y. But this is a contradiction since $U \neq X$.*

Definition 6 *A topological space (X, τ) is* α-symmetric *if for x and y in X, $x \in Cl_\alpha(\{y\})$ implies $y \in Cl_\alpha(\{x\})$.*

Definition 7 *A subset A of a topological space (X, τ) is called a* (α, α)-generalized-closed *set [21] (briefly (α, α)-g-closed) if $Cl_\alpha(A) \subset U$ whenever $A \subset U$ and U is α-open in (X, τ).*

Lemma 3.11 *Every α-closed set is (α, α)-g-closed.*

Theorem 3.12 *A topological space (X, τ) is α-symmetric if and only if $\{x\}$ is (α, α)-g-closed for each $x \in X$.*

Proof. Assume that $x \in Cl_\alpha(\{y\})$ but $y \notin Cl_\alpha(\{x\})$. This means that $[Cl_\alpha(\{x\})]^c$ contains y. Therefore the set $\{y\}$ is a subset of $[Cl_\alpha(\{x\})]^c$. This implies that $Cl_\alpha(\{y\})$ is a subset of $[Cl_\alpha(\{x\})]^c$. Now $[Cl_\alpha(\{x\})]^c$ contains x which is a contradiction.

Conversely, suppose that $\{x\} \subset E \in \alpha O(X, \tau)$ but $Cl_\alpha(\{x\})$ is not a subset of E. This means that $Cl_\alpha(\{x\})$ and E^c are not disjoint. Let $y \in Cl_\alpha(\{x\}) \cap E^c$. We have $x \in Cl_\alpha(\{y\})$ which is a subset of E^c and $x \notin E$. But this is a contradiction.

Corollary 3.13 *If a topological space (X, τ) is an α-T_1 space, then it is α-symmetric.*

Proof. In an α-T_1 space, singleton sets are α-closed (Theorem 3.8) and therefore (α, α)-g-closed (Lemma 3.11). By Theorem 3.1, the space is α-symmetric.

Corollary 3.14 *For a topological space (X, τ) the following are equivalent:*
(1) (X, τ) is α-symmetric and α-T_0;
(2) (X, τ) is α-T_1.

Proof. By Corollary 3.13 and Remark 3.3 it suffices to prove only (1) $\rightarrow$ (2). Let $x \neq y$ and by α-T_0, we may assume that $x \in G_1 \subset \{y\}^c$ for some $G_1 \in \alpha O(X, \tau)$. Then $x \notin Cl_\alpha(\{y\})$ and hence $y \notin Cl_\alpha(\{x\})$. There exists a $G_2 \in \alpha O(X, \tau)$ such that $y \in G_2 \subset \{x\}^c$. Hence (X, τ) is an α-T_1 space.

Definition 8 *A space (X, τ) is said to be $(\alpha, \alpha) - T_{\frac{1}{2}}$ if every (α, α)-g-closed set of X is α-closed.*

Theorem 3.15 *For an α-symmetric topological space (X, τ) the following are equivalent:*
(1) (X, τ) is α-T_0;
(2) (X, τ) is α-D_1;
(3 (X, τ) is (α, α)-$T_{\frac{1}{2}}$;
(4) (X, τ) is α-T_1.

The proof is straightforward and hence omitted.

We recall the following.

Definition 9 *A function $f : (X, \tau) \to (Y, \sigma)$ is α-irresolute [18] if the inverse image of each α-open set is α-open.*

Theorem 3.16 *If $f : (X, \tau) \to (Y, \sigma)$ is an α-irresolute surjective function and E is an αD-set in Y, then the inverse image of E is an αD-set in X.*

Proof. Let E be an αD-set in Y. Then there are α-open sets U_1 and U_2 in Y such that $S = U_1 \backslash U_2$ and $U_1 \neq Y$. By the α- irresoluteness of f, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are α-open in X. Since $U_1 \neq Y$, we have $f^{-1}(U_1) \neq X$. Hence $f^{-1}(E) = f^{-1}(U_1) \backslash f^{-1}(U_2)$ is an αD-set.

Theorem 3.17 *If (Y, σ) is α-D_1 and $f : (X, \tau) \to (Y, \sigma)$ is α-irresolute and bijective, then (X, τ) is α-D_1.*

Proof. Suppose that Y is an α-D_1 space. Let x and y be any pair of distinct points in X. Since f is injective and Y is α-D_1, there exist αD-sets G_x and G_y of Y containing $f(x)$ and $f(y)$ respectively, such that $f(y) \notin G_x$ and $f(x) \notin G_y$. By Theorem 3.16 , $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are αD-sets in X containing x and y, respectively. This implies that X is an α-D_1 space.

Recall that a map is *always α-open* [11] if the image of every α-open set is α-open.

Theorem 3.18 *Let X be an arbitrary space, R an equivalence relation in X and $p : X \to X/R$ the identification map. If $R \subset X \times X$ is α-closed in $X \times X$ and p is an always α-open map, then X/R is α-T_2.*

Proof. Let $p(x)$, $p(y)$ be distinct members of X/R. Since x and y are not related, $R \subset X \times X$ is α-closed in $X \times X$. There are α-open sets U and V such that $x \in U$, $y \in V$ and $U \times V \subset R^c$. Thus $p(U)$, $p(V)$ are disjoint and also α-open in X/R since p is always α-open.

Theorem 3.19 *A topological space (X, τ) is α-D_1 if and only if for each pair of distinct points $x, y \in X$, there exists an α-irresolute surjective function $f : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is an α-D_1 space such that $f(x)$ and $f(y)$ are distinct.*

Proof. Necessity. For every pair of distinct points of X, it suffices to take the identity function on X.
Sufficiency. Let x and y be any pair of distinct points in X. By hypothesis, there exists an α-irresolute, surjective function f of a space (X,τ) onto an α-D_1 space (Y,σ) such that $f(x) \neq f(y)$. Therefore, there exist disjoint αD-sets G_x and G_y in Y such that $f(x) \in G_x$ and $f(y) \in G_y$. Since f is α-irresolute and surjective, by Theorem 3.16, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint αD-sets in X containing x and y, respectively. Therefore the space X is an α-D_1 space.

Theorem 3.20 *The following four properties are equivalent:*
(1) X is α-T_2;
(2) Let $x \in X$. For each $y \neq x$, there exists an α-open set U such that $x \in U$ and $y \notin Cl_\alpha(U)$;
(3) For each $x \in X$, $\cap\{Cl_\alpha(U)/U \in \alpha O(X,\tau)$ and $x \in U\} = \{x\}$;
(4) The diagonal $\Delta = \{(x,x)/x \in X\}$ is α-closed in $X \times X$.

Proof. (1) $\rightarrow$ (2). Let $x \in X$ and $y \neq x$. Then there are disjoint α-open sets U and V such that $x \in U$ and $y \in V$. Clearly, V^c is α-closed, $Cl_\alpha(U) \subset V^c$, $y \notin V^c$ and therefore $y \notin Cl_\alpha(U)$.
(2) $\rightarrow$ (3). If $y \neq x$, then there exists an α-open set U such that $x \in U$ and $y \notin Cl_\alpha(U)$. So $y \notin \cap\{Cl_\alpha(U)/U \in \alpha O(X,\tau)$ and $x \in U\}$.
(3) $\rightarrow$ (4). We prove that Δ^c is α-open. Let $(x,y) \notin \Delta$. Then $y \neq x$ and since $\cap\{Cl_\alpha(U)/U \in \alpha O(X,\tau)$ and $x \in U\} = \{x\}$ there is some $U \in \alpha O(X,\tau)$ with $x \in U$ and $y \notin Cl_\alpha(U)$. Since $U \cap (Cl_\alpha(U))^c = \emptyset$, $U \times (Cl_\alpha(U))^c$ is an α-open set such that $(x,y) \in U \times (Cl_\alpha(U))^c \subset \Delta^c$.
(4) $\rightarrow$ (1). If $y \neq x$, then $(x,y) \notin \Delta$ and thus there exist α-open sets U and V such that $(x,y) \in U \times V$ and $(U \times V) \cap \Delta = \emptyset$. Clearly, for the α-open sets U and V we have: $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

4. α-R_0 spaces and α-R_1 spaces

Definition 10 *Let A be a subset of a topological space X. The α-kernel of A [16], denoted by $Ker_\alpha(A)$ is defined to be the set $Ker_\alpha(A) = \cap\{O \in \alpha O(X,\tau) \mid A \subset O\}$.*

Definition 11 *Let x be a point of a topological space X. The α-kernel of x, denoted by $Ker_\alpha(\{x\})$ is defined to be the set $Ker_\alpha(\{x\}) = \cap\{O \in \alpha O(X,\tau) \mid x \in O\}$.*

Lemma 4.1 *Let (X,τ) be a topological space and $x \in X$. Then*
$Ker_\alpha(A) = \{x \in X/Cl_\alpha(\{x\}) \cap A \neq \emptyset\}$.

Proof. Let $x \in Ker_\alpha(A)$ and $Cl_\alpha(\{x\}) \cap A = \emptyset$. Hence $x \notin X - Cl_\alpha(\{x\})$ which is an α-open set containing A. This is impossible, since $x \in Ker_\alpha(A)$. Consequently, $Cl_\alpha(\{x\}) \cap A \neq \emptyset$. Let $Cl_\alpha(\{x\}) \cap A \neq \emptyset$ and $x \notin Ker_\alpha(A)$. Then, there exists an α-open set D containing A and $x \notin D$. Let $y \in Cl_\alpha(\{x\}) \cap A$. Hence, D is an α-neighborhood of y which $x \notin D$. By this contradiction, $x \in Ker_\alpha(A)$ and the claim.

Definition 12 *A topological space (X, τ) is said to be an α-R_0 space if every α-open set contains the α-closure of each of its singletons.*

It is clear that α-R_0 implies pre-R_0 and α-R_0 implies semi-R_0 but the converses are not true.

Example 4.2 *Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{a, b\}, X\}$. This is pre-$R_0$ but not α-R_0 since $Cl_\alpha(\{a\}) = X \not\subset \{a, b\}$ and also not semi-R_0.*

Example 4.3 *Let $X = \{a, b, c\}$ be endowed with the topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then the space X is semi-R_0 but it is not pre-R_0 since $Cl(Int(\{a\}) \not\subset \{a, c\}$ where $\{a, c\} \in PO(X, \tau)$ [5]. Observe also that (X, τ) is not α-R_0.*

Remark 4.4 *Pre-R_0 and semi-R_0 spaces are independent. Example 4.3 is semi-R_0 but not pre-R_0 whereas in Example 4.2, the space X is pre-R_0 but not semi-R_0.*

Note that none of the implications in the diagram below is reversible.

$$\begin{array}{ccccc} R_0 & \rightarrow & \alpha\text{-}R_0 & \rightarrow & \text{semi-}R_0 \\ & & & \searrow & \\ & & & & \text{pre-}R_0 \end{array}$$

The notion of α-R_0 does not imply the notion of R_0 as it is shown by the following example.

Example 4.5 *Let p be a fixed point of (X, τ) with τ as the cofinite topology on X, i.e., $\tau = \{\emptyset, G, X\}$ with $G \subset X - p$ and $X - G$ finite. We can see that X is not R_0 , since if G is an open set and $x \in G$, then $Cl(x) = X \not\subset G$. But X is α-R_0 since X is α-T_1 and therefore every α-T_1 is α-R_0.*

Lemma 4.6 *Let (X, τ) be a topological space and $x \in X$. Then $y \in Ker_\alpha(\{x\})$ if and only if $x \in Cl_\alpha(\{y\})$.*

Proof. Suppose that $y \notin Ker_\alpha(\{x\})$. Then there exists an α-open set V containing x such that $y \notin V$. Therefore we have $x \notin Cl_\alpha(\{y\})$. The proof of the converse case can be done similarly.

Lemma 4.7 *The following statements are equivalent for any points x and y in a topological space (X, τ) :*
(1) $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$;
(2) $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$.

Proof. (1) $\rightarrow$ (2) : Suppose that $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$, then there exists a point z in X such that $z \in Ker_\alpha(\{x\})$ and $z \notin Ker_\alpha(\{y\})$. It follows from $z \in Ker_\alpha(\{x\})$ that $\{x\} \cap Cl_\alpha(\{z\}) \neq \emptyset$. This implies that $x \in Cl_\alpha(\{z\})$. By $z \notin Ker_\alpha(\{y\})$, we have $\{y\} \cap Cl_\alpha(\{z\}) = \emptyset$. Since $x \in Cl_\alpha(\{z\})$, $Cl_\alpha(\{x\}) \subset Cl_\alpha(\{z\})$ and $\{y\} \cap Cl_\alpha(\{x\}) = \emptyset$. Therefore, $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. Now $Ker_\alpha(\{x\}) \neq$

$Ker_\alpha(\{y\})$ implies that $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$.
$(2) \to (1)$: Suppose that $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. Then there exists a point $z \in X$ such that $z \in Cl_\alpha(\{x\})$ and $z \notin Cl_\alpha(\{y\})$. Then, there exists an α-open set containing z and therefore x but not y, i.e., $y \notin Ker_\alpha(\{x\})$. Hence $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$.

Theorem 4.8 *A topological space (X, τ) is an α-R_0 space if and only if for any x and y in X, $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$ implies $Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\}) = \emptyset$.*

Proof. Necessity. Suppose that (X, τ) is α-R_0 and $x, y \in X$ such that $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. Then, there exist $z \in Cl_\alpha(\{x\})$ such that $z \notin Cl_\alpha(\{y\})$ (or $z \in Cl_\alpha(\{y\})$) such that $z \notin Cl_\alpha(\{x\})$. There exists $V \in \alpha O(X, \tau)$ such that $y \notin V$ and $z \in V$; hence $x \in V$. Therefore, we have $x \notin Cl_\alpha(\{y\})$. Thus $x \in X - Cl_\alpha(\{y\}) \in \alpha O(X, \tau)$, which implies $Cl_\alpha(\{x\}) \subset X - Cl_\alpha(\{y\})$ and $Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\}) = \emptyset$. The proof for otherwise is similar

Sufficiency. Let $V \in \alpha O(X, \tau)$ and let $x \in V$. We will show that $Cl_\alpha(\{x\}) \subset V$. Let $y \notin V$, i.e., $y \in X - V$. Then $x \neq y$ and $x \notin Cl_\alpha(\{y\})$. This shows that $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. By assumption ,$Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\}) = \emptyset$. Hence $y \notin Cl_\alpha(\{x\})$ and therefore $Cl_\alpha(\{x\}) \subset V$.

Theorem 4.9 *A topological space (X, τ) is an α-R_0 space if and only if for any points x and y in X , $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$ implies $Ker_\alpha(\{x\}) \cap Ker_\alpha(\{y\}) = \emptyset$.*

Proof. Suppose that (X, τ) is an α-R_0 space. Thus by Lemma 4.7, for any points x and y in X if $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$ then $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. Now we prove that $Ker_\alpha(\{x\}) \cap Ker_\alpha(\{y\}) = \emptyset$. Assume that $z \in Ker_\alpha(\{x\}) \cap Ker_\alpha(\{y\})$. By $z \in Ker_\alpha(\{x\})$ and Lemma 4.6, it follows that $x \in Cl_\alpha(\{z\})$. Since $x \in Cl_\alpha(\{x\})$, by Theorem 4.8 $Cl_\alpha(\{x\}) = Cl_\alpha(\{z\})$. Similarly, we have $Cl_\alpha(\{y\}) = Cl_\alpha(\{z\}) = Cl_\alpha(\{x\})$. This is a contradiction. Therefore, we have $Ker_\alpha(\{x\}) \cap Ker_\alpha(\{y\}) = \emptyset$. Conversely, let (X, τ) be a topological space such that for any points x and y in X, $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$ implies $Ker_\alpha(\{x\}) \cap Ker_\alpha(\{y\}) = \emptyset$. If $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$, then by Lemma 4.7, $Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$. Hence $Ker_\alpha(\{x\}) \cap Ker_\alpha(\{y\}) = \emptyset$ which implies $Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\}) = \emptyset$. Because $z \in Cl_\alpha(\{x\})$ implies that $x \in Ker_\alpha(\{z\})$. Therefore $Ker_\alpha(\{x\}) \cap Ker_\alpha(\{z\}) \neq \emptyset$. By hypothesis, we have $Ker_\alpha(\{x\}) = Ker_\alpha(\{z\})$. Then $z \in Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\})$ implies that $Ker_\alpha(\{x\}) = Ker_\alpha(\{z\}) = Ker_\alpha(\{y\})$. This is a contradiction. Hence, $Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\}) = \emptyset$. By Theorem 4.8 (X, τ) is an α-R_0 space.

Theorem 4.10 *For a topological space (X, τ), the following properties are equivalent :*
(1) (X, τ) is an α-R_0 space;
(2) For any $A \neq \emptyset$ and $G \in \alpha O(X, \tau)$ such that $A \cap G \neq \emptyset$, there exists $F \in \alpha C(X, \tau)$ such that $A \cap F \neq \emptyset$ and $F \subset G$;
(3) Any $G \in \alpha O(X, \tau)$, $G = \cup\{F \in \alpha C(X, \tau) \mid F \subset G\}$;
(4) Any $F \in \alpha C(X, \tau)$, $F = \cap\{G \in \alpha O(X, \tau) \mid F \subset G\}$;
(5) For any $x \in X$, $Cl_\alpha(\{x\}) \subset Ker_\alpha(\{x\})$.

Proof. (1) → (2) : Let A be a nonempty set of X and $G \in \alpha O(X,\tau)$ such that $A \cap G \neq \emptyset$. There exists $x \in A \cap G$. Since $x \in G \in \alpha O(X,\tau), Cl_\alpha(\{x\}) \subset G$. Set $F = Cl_\alpha(\{x\})$, then $F \in \alpha C(X,\tau), F \subset G$ and $A \cap F \neq \emptyset$.
(2) → (3) : Let $G \in \alpha O(X,\tau)$, then $G \supset \cup\{F \in \alpha C(X,\tau) \mid F \subset G\}$. Let x be any point of G. There exists $F \in \alpha C(X,\tau)$ such that $x \in F$ and $F \subset G$. Therefore, we have $x \in F \subset \cup\{F \in \alpha C(X,\tau) \mid F \subset G\}$ and hence $G = \cup\{F \in \alpha C(X,\tau) \mid F \subset G\}$.
(3) → (4) : This is obvious.
(4) → (5) : Let x be any point of X and $y \notin Ker_\alpha(\{x\})$. There exists $V \in \alpha O(X,\tau)$ such that $x \in V$ and $y \notin V$; hence $Cl_\alpha(\{y\}) \cap V = \emptyset$. By (4) $(\cap\{G \in \alpha O(X,\tau) \mid Cl_\alpha(\{y\}) \subset G\}) \cap V = \emptyset$. There exists $G \in \alpha O(X,\tau)$ such that $x \notin G$ and $Cl_\alpha(\{y\}) \subset G$. Therefore, $Cl_\alpha(\{x\}) \cap G = \emptyset$ and $y \notin Cl_\alpha(\{x\})$. Consequently, we obtain $Cl_\alpha(\{x\}) \subset Ker_\alpha(\{x\})$.
(5) → (1) : Let $G \in \alpha O(X,\tau)$ and $x \in G$. Suppose $y \in Ker_\alpha(\{x\})$, then $x \in Cl_\alpha(\{y\})$ and $y \in G$. This implies that $Cl_\alpha(\{x\}) \subset Ker_\alpha(\{x\}) \subset G$. Therefore, (X,τ) is an α-R_0 space.

Corollary 4.11 *For a topological space (X,τ), the following properties are equivalent :*
(1) (X,τ) is an α-R_0 space;
(2) $Cl_\alpha(\{x\}) = Ker_\alpha(\{x\})$ for all $x \in X$.

Proof. (1) → (2) : Suppose that (X,τ) is an α-R_0 space. By Theorem 4.10, $Cl_\alpha(\{x\}) \subset Ker_\alpha(\{x\})$ for each $x \in X$. Let $y \in Ker_\alpha(\{x\})$, then $x \in Cl_\alpha(\{y\})$ and so $Cl_\alpha(\{x\}) = Cl_\alpha(\{y\})$. Therefore, $y \in Cl_\alpha(\{x\})$ and hence $Ker_\alpha(\{x\}) \subset Cl_\alpha(\{x\})$. This shows that $Cl_\alpha(\{x\}) = Ker_\alpha(\{x\})$.
(2) → (1) : This is obvious by Theorem 4.9

Theorem 4.12 *For a topological space (X,τ), the following properties are equivalent :*
(1) (X,τ) is an α-R_0 space;
(2) $x \in Cl_\alpha(\{y\})$ if and only if $y \in Cl_\alpha(\{x\})$, for any points x and y in X.

Proof. (1) → (2) : Assume that X is α-R_0. Let $x \in Cl_\alpha(\{y\})$ and D be any α-open set such that $y \in D$. Now by hypothesis, $x \in D$. Therefore, every α-open set containing y contains x. Hence $y \in Cl_\alpha(\{x\})$.
(2) → (1) : Let U be an α-open set and $x \in U$. If $y \notin U$, then $x \notin Cl_\alpha(\{y\})$ and hence $y \notin Cl_\alpha(\{x\})$. This implies that $Cl_\alpha(\{x\}) \subset U$. Hence (X,τ) is α-R_0.

We observed that by Definition 6 and Theorem 4.12 the notions of α-symmetric and α-R_0 are equivalent.

Theorem 4.13 *For a topological space (X,τ), the following properties are equivalent :*
(1) (X,τ) is an α-R_0 space;
(2) If F is α-closed, then $F = Ker_\alpha(F)$;

(3) If F is α-closed and $x \in F$, then $Ker_\alpha(\{x\}) \subset F$;
(4) If $x \in X$, then $Ker_\alpha(\{x\}) \subset Cl_\alpha(\{x\})$.

Proof. (1) $\to$ (2) : Let F be α-closed and $x \notin F$. Thus $X - F$ is α-open and contains x. Since (X, τ) is α-R_0, $Cl_\alpha(\{x\}) \subset X - F$. Thus $Cl_\alpha(\{x\}) \cap F = \emptyset$ and by Lemma 4.1 $x \notin Ker_\alpha(F)$. Therefore $Ker_\alpha(F) = F$.
(2) $\to$ (3) : In general, $A \subset B$ implies $Ker_\alpha(A) \subset Ker_\alpha(B)$. Therefore, it follows from (2) that $Ker_\alpha(\{x\}) \subset Ker_\alpha(F) = F$.
(3) $\to$ (4) : Since $x \in Cl_\alpha(\{x\})$ and $Cl_\alpha(\{x\})$ is α-closed, by (3) $Ker_\alpha(\{x\}) \subset Cl_\alpha(\{x\})$.
(4) $\to$ (1) : We show the implication by using Theorem 4.10. Let $x \in Cl_\alpha(\{y\})$. Then by Lemma 4.6, $y \in Ker_\alpha(\{x\})$. Since $x \in Cl_\alpha(\{x\})$ and $Cl_\alpha(\{x\})$ is α-closed, by (4) we obtain $y \in Ker_\alpha(\{x\}) \subset Cl_\alpha(\{x\})$. Therefore $x \in Cl_\alpha(\{y\})$ implies $y \in Cl_\alpha(\{x\})$. The converse is obvious and (X, τ) is α-R_0.

Recall that a filterbase F is called *α-convergent* [11] to a point x in X, if for any α-open set U of X containing x, there exists B in F such that B is a subset of U.

Lemma 4.14 *Let (X, τ) be a topological space and x and y any two points in X such that every net in X α-converging to y α-converges to x. Then $x \in Cl_\alpha(\{y\})$.*

Proof. Suppose that $x_n = y$ for each $n \in \mathbf{N}$. Then $\{x_n\}_{n\in\mathbf{N}}$ is a net in $Cl_\alpha(\{y\})$. Since $\{x_n\}_{n\in\mathbf{N}}$ α-converges to y , then $\{x_n\}_{n\in\mathbf{N}}$ α-converges to x and this implies that $x \in Cl_\alpha(\{y\})$.

Theorem 4.15 *For a topological space (X, τ), the following statements are equivalent :*
(1) (X, τ) is an α-R_0 space;
(2) If $x, y \in X$, then $y \in Cl_\alpha(\{x\})$ if and only if every net in X α-converging to y α-converges to x.

Proof. (1) $\to$ (2) : Let $x, y \in X$ such that $y \in Cl_\alpha(\{x\})$. Suppose that $\{x_\alpha\}_{\alpha\in N}$ is a net in X such that $\{x_\alpha\}_{\alpha\in N}$ α-converges to y. Since $y \in Cl_\alpha(\{x\})$, by Theorem 4.8 we have $Cl_\alpha(\{x\}) = Cl_\alpha(\{y\})$. Therefore $x \in Cl_\alpha(\{y\})$. This means that $\{x_\alpha\}_{\alpha\in\Lambda}$ α-converges to x. Conversely, let $x, y \in X$ such that every net in X α-converging to y α-converges to x. Then $x \in Cl_\alpha(\{y\})$ by Lemma 4.1. By Theorem 4.8, we have $Cl_\alpha(\{x\}) = Cl_\alpha(\{y\})$. Therefore $y \in Cl_\alpha(\{x\})$.
(2) $\to$ (1) : Assume that x and y are any two points of X such that $Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\}) \neq \emptyset$. Let $z \in Cl_\alpha(\{x\}) \cap Cl_\alpha(\{y\})$. So there exists a net $\{x_\alpha\}_{\alpha\in\Lambda}$ in $Cl_\alpha(\{x\})$ such that $\{x_\alpha\}_{\alpha\in\Lambda}$ α-converges to z. Since $z \in Cl_\alpha(\{y\})$, then $\{x_\alpha\}_{\alpha\in\Lambda}$ α-converges to y. It follows that $y \in Cl_\alpha(\{x\})$. Similarly we obtain $x \in Cl_\alpha(\{y\})$. Therefore $Cl_\alpha(\{x\}) = Cl_\alpha(\{y\})$ and by Theorem 4.8, (X, τ) is α-R_0.

Definition 13 *A topological space (X, τ) is said to be α-R_1 if for x, y in X with $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$, there exist disjoint α-open sets U and V such that $Cl_\alpha(\{x\})$ is a subset of U and $Cl_\alpha(\{y\})$ is a subset of V.*

Clearly every α-R_1 space is α-R_0. Indeed let U be an α-open such that $x \in U$. If $y \notin U$, then since $x \notin Cl_\alpha(\{y\})$, $Cl_\alpha(\{x\}) \neq Cl_\alpha(\{y\})$. Hence, there exists an α-open V_y such that $Cl_\alpha(\{y\}) \subset V_y$ and $x \notin V_y$, which implies $y \notin Cl_\alpha(\{x\})$. Thus $Cl_\alpha(\{x\}) \subset U$. Therefore (X, τ) is α-R_0.

Example 4.16 *Let p be a fixed point of (X, τ) with τ as the cofinite topology on X, i.e., $\tau = \{\emptyset, G, X\}$ with $G \subset X - p$ and $X - G$ finite. The space X is α-R_0 but it is not α-R_1.*

Theorem 4.17 *A topological space (X, τ) is α-R_1 if and only if for $x, y \in X, Ker_\alpha(\{x\}) \neq Ker_\alpha(\{y\})$, there exist disjoint α-open sets U and V such that $Cl_\alpha(\{x\}) \subset U$ and $Cl_\alpha(\{y\}) \subset V$.*

Proof. It follows from Lemma 4.6.

5. Weakly α-R_0 spaces

Definition 14 *A topological space (X, τ) is said to be* weakly α-R_0 *if $\cap_{x \in X} Cl_\alpha(\{x\}) = \emptyset$.*

Theorem 5.1 *A topological space (X, τ) is weakly α-R_0 if and only if $Ker_\alpha(\{x\}) \neq X$ for every $x \in X$.*

Proof. Suppose that the space (X, τ) is weakly α-R_0. Assume that there is a point y in X such that $Ker_\alpha(\{y\}) = X$. Then $y \notin O$, where O is some proper α-open subset of X. This implies that $y \in \cap_{x \in X} Cl_\alpha(\{x\})$. But this is a contradiction.

Now assume that $Ker_\alpha(\{x\}) \neq X$ for every $x \in X$. If there exists a point $y \in X$ such that $y \in \cap_{x \in X} Cl_\alpha(\{x\})$, then every α-open set containing y must contain every point of X. This implies that the space X is the unique α-open set containing y. Hence $Ker_\alpha(\{x\}) = X$ which is a contradiction. Therefore, (X, τ) is weakly α-R_0.

Remark 5.2 *It should be noted that since $sCl(\{x\}) \subset Cl_\alpha(\{x\}) \subset Cl(\{x\})$ and $pCl(\{x\}) \subset Cl_\alpha(\{x\}) \subset Cl(\{x\})$, we have the following diagram in which the converses of the implications are not true.*

Weakly R_0 $\rightarrow$ Weakly α-R_0 $\rightarrow$ Weakly semi-R_0
$\searrow$
Weakly pre-R_0

In [15], it is shown that every weakly R_0 space is weakly pre-R_0 ([15], Theorem 2.1) and the converse is not true ([15], Example 2.1). Moreover it is shown that the notions of weakly pre-R_0 and weakly semi-R_0 are independent of each other ([15], Example 2.2 and Example 2.3). Also ([15], Example 2.2) and ([15], Example 2.23) show that weakly pre-R_0 and wealy semi-R_0 do not imply weakly α-R_0.

Definition 15 *A function $f : X \to Y$ is called always α-closed if the image of every α-closed subset of X is α-closed in Y.*

Theorem 5.3 *If $f : X \to Y$ is an injective always α-closed function and X is weakly α-R_0, then Y is weakly α-R_0.*

Proof. Straightforward.

Theorem 5.4 *If the topological space X is weakly α-R_0 and Y is any topological space, then the product $X \times Y$ is weakly α-R_0.*

Proof. If we show that $\cap_{(x,y)\in X\times Y} Cl_\alpha(\{x,y\}) = \emptyset$, then we are done. Observe that $\cap_{(x,y)\in X\times Y} Cl_\alpha(\{x,y\}) \subset \cap_{(x,y)\in X\times Y}(Cl_\alpha(\{x\})\times Cl_\alpha(\{y\})) = \cap_{x\in X} Cl_\alpha(\{x\})\times \cap_{y\in Y} Cl_\alpha(\{y\}) \subset \emptyset \times Y = \emptyset$ and hence the proof.

Acknowledgements. The authors are grateful to the referee for his valuable work which improved the quality of this paper.

References

1. D. Andrijevic, Some properties of the topology of α-sets, *Mat. Vesnik* **36** (1984), 1-10.
2. F. G. Arenas, J. Cao, J. Dontchev and M. L. Puertas, Some covering properties of the α-topology (preprint).
3. S. P. Arya and T. M. Nour, Weakly semi-R_0 spaces, *Indian J. Pure Appl. Math.*, **2** (12) (1990), 1083-85.
4. M. Caldas, D. N. Georgiou and S. Jafari, Study of (Λ, α)-closed sets and the related notions in topological spaces (submitted).
5. M. Caldas, S. Jafari and T. Noiri, Characterizations of pre-R_0 and pre-R_1, *Topology Proceedings*, **25**(2000), 17-30.
6. M. Caldas and J. Dontchev, On spaces with hereditarily compact α-topologies, *Acta Math. Hung.* **82** (1999), 121-129.
7. M. Caldas, A separation axiom between semi-T_0 and semi-T_1, *Mem. Fac. Sci. Kochi Univ. (Math.)* **18** (1997), 37-42.
8. S. G. Crossley and S. K. Hildebrand, Semi-closure, *Texas J. Sci.* **22** (1971), 99-112.
9. S. G. Crossley and S. K. Hildebrand, Semi-topological properties, *Fund. Math.* **74** (1972), 233-254.
10. J. Di Maio, A separation axiom weaker than R_0, *Indian J. Pure Appl. Math.* **16**(4) (1985), 373-375
11. S. Jafari, Rare α-continuity (submitted).
12. S. Jafari, On a weak separation axiom, Far East J. Math. Sci. (to appear).
13. S. Jafari and T. Noiri, Contra-α-continuous functions between topological spaces, *Iranian Int. J. Sci.* **2**(2)(2001),153-167.
14. S. Jafari and T. Noiri, Some remarks on weak α-continuity, *Far East J. Math. Sci.* **6**(4) (1998), 619-625.
15. S. Jafari, A note on weakly pre-R_0 spaces, *Far East J. Math. Sci.* **3**(1) (2001), 93-96.
16. S. Jafari and T. Noiri, Contra-strongly α-irresolute functions (preprint).

17. N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly* **70** (1963), 36-41.

18. S.N. Maheshwari and S.S. Thakur, On α-irresolute mappings, *Tamkang J. Math.* **11** (1980), 209-214.

19. S.N. Maheshwari and S.S. Thakur, On α-compact spaces, *Bull. Inst. Math. Acad. Sinica* **13** (1985), 341-347.

20. S.N. Maheshwari and R. Prasad, On $(R_0)_s$-spaces, *Portugaliae Math.* **34** (1975), 213-217.

21. H. Maki, R. Devi and K. Balachandran, Generalized α-closed sets in topology, *Bull. Fukuoka Univ. Ed.* Part III, **42** (1993), 13-21.

22. H. Maki and T. Noiri, The pasting lemma for α-continuous maps, *Glas. Mat.* **23**(43) (1988), 357-363.

23. A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, α-continuous and α-open mappings, *Acta Math. Hung.* **41** (1983), 213-218.

24. A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc. Egypt* **51** (1982), 47-53.

25. O. Njåstad, On some classes of nearly open sets, *Pacific J. Math.* **15** (1965), 961-970.

26. T. Noiri and G. Di Maio, Properties of α-compact spaces, *Suppl. Rend. Circ. Mat. Palermo* (2) 18 (1988), 359-369.

27. T. Noiri, S. N. El-Deeb, I. A. Hasanein and A. S. Mashhour, On p-regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie* **27** (**25**) (1983), 311-315.

28. T. Noiri, Weakly α-continuous functions, *Internat. J. Math. Math. Sci.* **10**(3) (1987), 483-490.

29. T. Noiri, On α-continuous functions, *Časopis Pěst. Mat.* **109** (1984), 118-126.

30. I. L. Reilly and M. K. Vamanamurthy, On α-continuity in topological spaces, *Acta math. Hung.* **45** (1-2) (1985), 27-32.

31. I. L. Reilly and M. K. Vamanamurthy, On α-sets in topological spaces, *Tamkang J. Math.* **16** (1985), 7-11.

32. I. L. Reilly and M. K. Vamanamurthy, On countably α-compact spaces, *J. Sci. Res.* **5** (1983), 5-8.

Addresses :

M. Caldas
Departamento de Matematica Aplicada,
Universidade Federal Fluminense,
Rua Mario Santos Braga, s/n
24020-140, Niteroi, RJ Brasil.
e-mail: gmamccs@vm.uff.br

D.N. Georgiou
Department of Mathematics,
University of Patras, 26500 Patras, Greece
e-mail: georgiou@math.upatras.gr

S. Jafari

Department of Mathematics and Physics,
Roskilde University,Postbox 260, 4000
Roskilde, Denmark.
e-mail: sjafari@ruc.dk